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Development of a Modified SMA Based MSCS-CN Model for Runoff Estimation

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Abstract The Soil Conservation Service Curve Number (SCS-CN) method developed by the USDA-Soil Conservation Service (SCS, 1972) is widely used for the estimation of direct runoff for a given rainfall event from small agricultural watersheds. The initial soil moisture plays an important role in re-structuring of the SCS-CN method and enables us to prevent unreasonable sudden jump in runoff estimation and this has prompted the concept of soil moisture accounting (SMA) procedure to develop improved SCS-CN based models. Applying the concept of SMA procedure and changed parameterization, Michel et al. Water Resour Res 41(2):1–6 (2005) developed an improved SCS-CN model (MSCS-CN), which could be thought of an improvement over the existing SCS-CN method; however, their model still inherits several conceptual limitations and inconsistencies. Therefore, in this study an attempt is made to propose an improved SMA based SCS-CN-inspired model (MMSCS-CN) model incorporating a continuous function for initial soil moisture and test its suitability over the MSCS-CN and SCS-CN model using a large dataset from US watersheds. Using, Nash and Sutcliffe efficiency (NSE) and root mean square error (RMSE) of these models, the overall performance is further evaluated using rank grading system, and it is found that the MMSCS-CN scores highest mark (95; overall rank I) followed by MSCS-CN with 61 (overall rank II), and SCS-CN model with 51 mark (overall rank III) out of the maximum 105. This study shows

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that the proposed MMSCS-CN model has several advantages and performs better than the MSCS-CN and the existing SCS-CN model.

1 Introduction

Estimation of surface runoff is essential for the assessment of water yield potential of watersheds, planning of soil and water conservation structures, reducing sedimentation, and downstream flooding hazards. Although many hydrologic models are available for the estimation of direct surface runoff from storm rainfall; however, most models have limited applicability because of their intensive input data needs and calibration requirements (Shi et al. 2009). The Soil Conservation Service Curve Number (SCS-CN) method, developed by the USDA-Soil Conservation Service (SCS 1972), has wider applicability for estimation of direct runoff for a given rainfall event from small agricultural watersheds (Soulis and Valiantzas 2013). The method is simple to use and only requires basic descriptive inputs that are converted into numeric values for estimation of watershed direct surface runoff volume (Bonta 1997). A curve number (CN) that is descriptive of major runoff producing characteristics of watershed such as soil type, land use/treatment classes, hydrologic soil group, hydrologic condition, most importantly the antecedent moisture condition (AMC) is required in the application of the method. The SCS-CN method is well suited for estimating surface runoff from small agricultural watersheds and establishes CN values under various antecedent moisture conditions with generally high practicability (Chung et al. 2010). As a result, the other modified models, despite their complicated forms, are unable to be directly applicable to real situations due to the problem of model closure, making the SCS-CN method one of the most popular models over the past several decades. According to Garen and Moore (2005), the reason for wide application of the SCS-CN method includes its simplicity, ease of use, widespread acceptance, and the significant infrastructure and institutional momentum for this procedure within Natural Resource Conservation Service. To date, there is no alternative that possesses as many advantages, which is why it has been and continues to be commonly used, whether or not it is, in a strict scientific sense, appropriate.

Despite appealing to many practicing hydrologists by its simplicity, the SCS-CN model contains some unknowns and inconsistencies (Chen 1982). The SCS-CN model suffers from an inconsistent unstable theoretical foundation and has several disadvantages, such as the inability to flexibly display antecedent moisture conditions, unaccounted rainfall intensity and storm duration, discrete unrealistic representation of CN and antecedent condition, and fixing of initial abstraction coefficient (λ) (Ponce and Hawkins 1996; De Michele and Salvadori 2002; Michel et al. 2005; Jain et al. 2006 and Shi et al. 2009). The fixing of λ (=0.2) has frequently been questioned for its validity and applicability invoking many researchers for a critical examination of the I_a–S relationship in pragmatic applications, where I_a = initial abstraction before surface runoff occurs and S = potential maximum retention (Hawkins et al. 2001). More recently, Ajmal et al. (2015) investigated SCS-CN and its inspired models for runoff estimation in South Korean watersheds using a dataset of 658 large storm-events and they found that lower values of the initial abstraction coefficient ($\lambda<0.2$) exhibited better runoff estimation than the fixed value, i.e., $\lambda=0.2$.

According to Michel et al. (2005) the empirical framework within which the SCS-CN method was developed that not incites hydrologists to explore the structural foundation and inherited limitations. They highlighted several structural inconsistencies, arising partly from the confusion between intrinsic parameters and initial conditions, and partly from an incorrect use of the underlying soil moisture accounting (SMA) procedure. The SMA procedure is based on the notion that higher the moisture store level, higher the fraction of rainfall that is converted into runoff. If the moisture storage level is full, all the rainfall will become runoff. Other than the information contained in National Engineering Handbook (NEH) section-4, which was not intended to be exhaustive, no complete account of the method's foundation is available to date, despite some noteworthy diagnostic attempts made by Chen (1982), Miller and Cronshey (1989), Ponce and Hawkins (1996), Mishra and Singh (1999), Mishra and Singh (2003), Michel et al. (2005), and Chung et al. (2010).

However, the method has always been a good choice among the hierarchy of the hydrological models, and has witnessed a myriad of applications all over the world throughout the entire spectrum of hydrology and water resources, even for problems it was not intended to solve such as such long-term hydrologic simulation (Williams and LaSeur 1976; Hawkins 1978; Knisel 1980; Choi et al. 2002; Mishra and Singh 2004; Michel et al. 2005; Sahu et al. 2010; Babu and Mishra 2012), metal partitioning and water quality (Mishra et al. 2004a; Ojha 2012), sediment yield modeling (Mishra et al. 2006a; Singh et al. 2008; Bhunya et al. 2010), and rainwater harvesting (Kadam et al. 2012; Singh et al. 2013). Recently, Singh et al. (2010) presented an updated hydrological review of the recent advancements in SCS-CN methodology and discussed its physical and mathematical significance in hydrological modelling.

Well known to the fact that the SCS-CN method exhibits variability in runoff computation due to spatial and temporal variability of rainfall, quality of measured runoff data, and the variability of antecedent rainfall and the antecedent moisture condition (AMC) (Ponce and Hawkins 1996). The AMC is categorized into three levels, i.e., AMC I (dry), AMC II (normal), and AMC III (wet), which statistically correspond, respectively, to 90, 50, and 10 % cumulative probability of exceedance of runoff depth for a given rainfall (Hjelmfelt et al. 1982). The three AMC levels permit unreasonable sudden jumps in curve numbers (CN), which result in corresponding jumps in estimated runoff. Mishra and Singh (2006) investigated the variation of CN with AMC conditions and developed power relationships between CN and 5-day antecedent rainfall amount to prevent the sudden jump from one AMC level to the other. This classical problem of sudden jump in SCS-CN method was well addressed by Michel et al. (2005) through incorporating the SMA concept, which led to the development of an improved version of SCS-CN model (hereafter named as Michel SCS-CN (MSCS-CN) model). In which they found that the SCS-CN method should feature upon initial soil moisture (V_0) condition rather than an unrealistic intrinsic parameter in form of initial abstractions (I_a). However, the improved model does not include any definite rules how to account for initial soil moisture (V_0) to prevent sudden jumps in runoff estimations, rather subjective adjustments are made for V_0 to accommodate all three AMCs in terms of and CN (or S) to retain all the simplicity and in all likelihood, the potential efficiency of the original SCS-CN model.

Therefore, keeping the aforementioned discussions in mind, this study proposes a modified MSCS-CN (MMSCS-CN) model based on revised SMA procedure for runoff estimation and suggest simple formulations for V_0 estimation. Thus, the major objectives of this study are twofold: (i) to formulate a modified MSCS-CN (MMSCS-CN) model based on revised SMA procedure with simplified formulations for V_0 estimation to avoid the sudden jump in V_0 and

AMC and (ii) to compare the performance of the proposed MMSCS-CN model with MSCS-CN model and the original SCS-CN method using a large field dataset of USA watersheds.

2 SCS-CN Method

The SCS-CN method mainly consists of the water balance equation and two fundamental hypotheses as:

$$P = Ia + F + Q, \tag{1}$$

$$\frac{Q}{P{-}Ia} = \frac{F}{S}, \tag{2}$$

$$Ia = \lambda S \tag{3}$$

where P = total rainfall, Q = actual amount of direct surface runoff Q, F = cumulative infiltration in soil when surface runoff occurs, I_a = initial abstraction before surface runoff occurs, λ = initial abstraction coefficient (dimensionless), and S = potential maximum retention, also described as the potential post initial abstraction retention (McCuen 2002). The values of P, Q, and S are given in depth dimensions. Combination of Eqs. (1) and (2) leads to the core equation of the SCS-CN as:

$$Q = \frac{(P{-}Ia)^2}{P{-}Ia + S}, \text{ if } P > I_a, Q = 0, \text{ otherwise} \tag{4}$$

Equation (4) is the general form of the popular SCS-CN method and is valid for $P \geq I_a$, $Q = 0$ otherwise. For $\lambda = 0.2$, the coupling of Eqs. (3) and (4) results in:

$$Q = \frac{(P{-}0.2S)^2}{P + 0.8S} \tag{5}$$

Equation (5) is the popular form of the existing SCS-CN method. Thus, the existing SCS-CN method with $\lambda = 0.2$ is a one-parameter model for computing surface runoff from daily storm rainfall. The parameter S of the SCS-CN method depends on soil type, land use, hydrologic condition, and AMC. The parameter S is mapped onto a dimensionless curve number CN, varying in a range $0 \leq CN \leq 100$, as:

$$S = \frac{25400}{CN} - 254 \tag{6}$$

where S has the unit in mm. The difference between S and CN is that the former is a dimensional quantity (L) whereas the later is non-dimensional. A CN=100 represents the condition of zero potential maximum retention (S=0), that is, an impermeable watershed. Conversely, CN=0 depicts a potential maximum retention (S = ∞), that is an infinitely abstracting watershed. The CN has no intrinsic meaning; it is only a convenient transformation of S to establish a 0–100 scale (Hawkins 1978).

3 Michel SCS-CN Model (MSCS-CN)

Michel et al. (2005) unveiled major inconsistencies associated with the SCS-CN model. A changed parameterization and sounder perception of underlying SMA procedure led to the development of an improved SCS-CN model (hereafter named as Michel SCS-CN (MSCS-CN) model). The general form of the MSCS-CN model can be expressed as (Michel et al. 2005):

$$V_0 \leq S_a - P \quad Q = 0 \tag{7}$$

$$S_a - P \; < \; V_0 < S_a \quad Q = \frac{(P + S_a - V_0)^2}{P + V_0 - S_a + S} \tag{8}$$

$$S_a \leq V_0 \leq S_a + S \qquad Q = P\left[1 - \frac{(S + S_a - V_0)^2}{S^2 + (S + S_a - V_0)P}\right] \tag{9}$$

where S_a = threshold soil moisture = $(V_0 + I_a)$, V_0 = initial soil moisture and I_a = Initial abstractions. The values of S_a and V_0 are given in depth dimensions The MSCS-CN model is both hydrologically more consistent and structurally stable, and retains all the simplicity and, in all likelihood, the potential of the original SCS-CN model. Notably, the MSCS-CN model does not have any expression for V_0 and S_a, however, it was suggested to identify typical situations where V_0 could be set in relation to the parameters S and S_a. Further, in an attempt to retain simplicity, subjective adjustments were made for V_0 in terms of S and letting $S_a = S/3$ to accommodate all the three AMCs, which could lead to quantum jump in V_0 and as a result in runoff estimation.

4 Why Modified MSCS-CN (MMSCS-CN) Model?

As discussed above, the MSCS-CN model was developed through changed parameterization and a more complete assessment of initial soil moisture (SMA) implied by the SCS-CN equation. Though the procedure is more consistent from SMA viewpoint and introduces initial soil moisture (V_0) and threshold soil moisture (S_a), that eliminates initial abstraction (Ia) to compute the direct surface runoff. However, if we critically examine the MSCS-CN model (Eqs. (7)–(9)), it can be easily realized that the model corresponding to the Eq. (8), entirely depends upon the existing SCS-CN method (Eq. 4), that omits the SMA procedure from its first fundamental hypothesis (Eq. 2) or C = S_r concept; where C = [Q/(P-Ia)] and S_r = F/S (Mishra and Singh 2002, 2003; Jain et al. 2006). Further, Eq. (4) has been consistently used with changed parameterization (i.e., replacing I_a through V_0 and S_a) for the development of the MSCS-CN procedure. Therefore, there exists a scope for further improvement in the MSCS-CN model.

5 Modified MSCS-CN Model (MMSCS-CN)

The modification was started from the original SCS-CN method (Eq. (4)) as follows. Here, two things are important to consider: (i) First fundamental hypothesis (Eq. (2)) does not explicitly

account for the initial soil moisture (V_0), which is the foundation of the existing SCS-CN method and (ii) Eq. (4) is hydrologically inconsistent as it does not yield $Q = 0$ for $P < I_a$. Therefore, to remove these structural inconsistencies and to account for V_0, the basic hypothesis (Eq. (2)) should be modified using $C=Sr$ concept (Mishra and Singh 2002, 2003; Mishra et al. 2004b), expressed as:

$$\frac{Q}{P-Ia} = \frac{F + V_0 + Ia}{S + V_0 + Ia} \tag{10}$$

Coupling of Eqs. (1) and (10) results into

$$\begin{aligned} Q &= \frac{(P + V_0)(P-Ia)}{P + S + V_0} \quad P > I_a \\ &= 0 \qquad\qquad\qquad \text{otherwise} \end{aligned} \tag{11}$$

Analytically, Eq. (11) is an improved form of the existing SCS-CN method and is derived after incorporating V_0 in $C = S_r$ concept. Since the condition $P < I_a$ yields a negative runoff, it should can be safely taken as zero.

Assuming that V_0 is the initial soil moisture storage, V is the soil moisture storage at any time t during a storm event, P is the accumulated rainfall up to the time t, Q is the corresponding runoff. Then the following expressions can be easily obtained as:

$$V = V_0 + (P-Q) \tag{12}$$

Differentiating Eq. (12) with time t yields

$$\frac{dV}{dt} = p-q \tag{13}$$

where $p = dP/dt$ and $q = dQ/dt$. Notably, V_0 was ignored in the original SCS-CN method. Putting Q from Eq. (11) into Eq. (12) yields an expression for V as:

$$V = V_0 + P - \left[\frac{(P + V_0)(P-Ia)}{P + S + V_0}\right] \tag{14a}$$

Differentiating Eq. (11) with respect to time t yields

$$q = p\left[\frac{(P + V_0 + S)(2P + V_0-Ia)-(P + V_0)(P-Ia)}{(P + S + V_0)^2}\right] \tag{14b}$$

The coupling of Eqs. (14a) and (14b) results into the following expression of q as (Appendix A):

$$q = p\left[\frac{VS + (V-Sa)(S_b-V)}{SSb}\right] \tag{15}$$

where S_b = absolute potential maximum retention = $(S+S_a)$, S, and S_a are constant for a given watershed and storm and have the dimensions of depth units. Equation (15) contains three terms in numerator viz., VS, $(V -S_a)$ and (S_b-V). If all these terms are positive, the equation yields a non-negative runoff q. As noted, VS and (S_b-V) are always non-negative. The second term $(V-S_a)$ may however take any value, positive or

negative, depending on $V > S_a$ or $V < S_a$. Now, putting $I_a = (S_a-V_0)$ into Eq. (11) results in

$$Q = \frac{(P + V_0)(P + V_0 - Sa)}{P + S + V_0} \quad \text{for } P + V_0 > S_a$$
$$= 0 \quad \text{otherwise} \tag{16}$$

If the soil is fully saturated before the start of storm event, i.e., $V_0 = S_b$, then Q should be equal to P from Eq. (16). However, putting $V_0 = S_b$ in Eq. (16) yields,

$$Q = P + \frac{SSb}{P + S + S_b} \tag{17}$$

Equation (17) shows that Q is greater than P and this implies that Eq. (16) is still not suitable for fully saturated condition (i.e., $V_0 = S_b$) as such and needs further refinement. This mathematical inconsistency can be removed through Eqs. (13) and (15) as follows:

$$\frac{dV}{dt} = p - p\left[\frac{VS + (V-Sa)(S_b-V)}{SSb}\right] \tag{18}$$

Integrating Eq. (18) with respect to t with appropriate upper and lower limits and substituting V from Eq. (13) results into the expression of Q and P as (Appendix B):

$$Q = P\left(1 - \frac{(S_b-V_0)^2}{SSb + P(S_b-V_0)}\right) \tag{19}$$

It is observed from Eq. (19) that if $V_0 = S_b$, then $Q = P$, and this is consistent both mathematically and physically. Similarly, for the lowest and intermediate conditions, i.e., (i) for $V0<Sa-P$ or $P < I_a$ i.e. rainfall P is not large enough to meet I_a requirement then $Q = 0$ and (ii) for $V0<Sa$, but $(P+V0)>Sa$ i.e. $Sa-P<V0<Sa$, then the generated runoff (Q) can be computed using Eq. (16).

In summary, these three conditions are given as:

$$V_0 < Sa-P \quad Q = 0 \tag{20}$$

$$Sa-P < V_0 < Sa \quad Q = \frac{(P + V_0)(P + V_0 - Sa)}{P + S + V_0} \tag{21}$$

$$Sa \leq V_0 \leq S_b \quad Q = P\left(1 - \frac{(S_b-V_0)^2}{SSb + P(S_b-V_0)}\right) \tag{22}$$

The above three equations represents the modified MSCS-CN model (MMSCS-CN) model. The MMSCS-CN represents a more rational and structurally more stable hydrological model as compared to the MSCS-CN model for surface runoff estimation. The applicability and efficacy of the MMSCS-CN model as compared to the MSCS-CN model and the original SCS-CN m.

6 Application of the Three Models

The applicability of the MMSCS-CN, MSCS-CN, and original SCS-CN model for runoff estimation was tested using a large rainfall-runoff data set from the US Department of Agriculture-Agricultural Research service (USDA-ARS) watershed data base. These data are collection of rainfall-runoff observations from small agricultural watersheds in the US. In the present study, rainfall-runoff data of 9359 events were taken from 35 watersheds having areas varying from 0.71 to 53.42 ha.

7 Estimation of the Initial Soil Moisture (V$_0$) and Threshold Soil Moisture (S$_a$)

As discussed above, the MSCS-CN model does not have any formulation for initial soil moisture (V$_0$). However, certain adjustments were suggested for V$_0$ to accommodate all three AMCs in terms of CN (or S). Therefore, in this study, a well-tested procedure as suggested by Mishra et al. (2006b) has been used for V$_0$ and S$_a$ computations as follows.

$$V_0 = \alpha \sqrt{P_5} S \tag{23a}$$

$$S_a = \beta S \tag{23b}$$

where α and β are coefficients and P$_5$ is the 5-day antecedent rainfall amount. The advantage of this expression is that it physically relates V$_0$ to P$_5$ and S, in the sense that a higher P$_5$ or S will give a higher V$_0$. Moreover, it obviates the sudden jump of V$_0$ with S or CN.

The coefficients α and β were determined using Marquardt constrained least-square (MCLS) approach (Marquardt 1963). For the original SCS-CN model, in all applications, the initial estimate of CN was taken as 50 and was allowed to vary from 0 to 100. Similarly, for the MMSCS-CN and MSCS-CN models, α was allowed to vary in the range of 0.01 to 2.0 with its initial value as 0.1. The parameter β was allowed to vary from 0.00 to 1.00 with an initial estimate of 0.10. However, for the MSCS-CN model, the parameter β was taken as 0.33. Summary statistics of computed values of models parameters are given in Table 1.

Table 1 Summary of the parameters resulting from application of the models to 35 watersheds

Sl. No.	Model/procedure	Parameter	Mean	Median	Min.	Max.	90 % Confidence interval	
							Lower	Higher
1	SCS-CN	S (mm) (for AMC II)	57.21	42.106	15.33	304.49	42.745	71.67
2	MSCS-CN	α	0.330	0.334	0.010	0.846	0.292	0.368
		S (mm)	68.07	57.12	5.69	364.33	52.14	83.72
3	MMSCS-CN	α	0.220	0.224	0.010	0.450	0.195	0.244
		β	0.118	0.104	0.001	0.386	0.088	0.149
		S (mm)	147.01	103.86	49.06	826.45	106.47	187.54

8 Performance Evaluation

The root mean square error (RMSE) and Nash and Sutcliffe (1970) efficiency (NSE) were taken as indices of agreement between computed and observed runoff

Table 2 Models performance based on RMSE and NSE resulted by applications of models to 35 watersheds

Sl. No.	Watershed ID	Area (ha)	MMSCS-CN model		MSCS-CN model		SCS-CN model	
			RMSE (mm)	NSE (%)	RMSE (mm)	NSE (%)	RMSE (mm)	NSE (%)
1	9004	23.96	1.97	76.63	2.05	74.88	2.97	49.094
2	16010	40.47	2.44	41.71	2.65	31.00	3.06	38.01
3	17001	11.02	5.78	78.77	5.86	78.15	12.30	75.24
4	17002	20.21	5.89	78.90	6.00	78.12	12.72	74.95
5	17003	5.08	5.22	74.83	5.18	75.15	10.51	65.7
6	26010	0.55	4.10	63.26	4.40	57.63	7.32	60.7
7	26013	0.68	3.69	34.57	3.99	23.20	4.77	31.49
8	26014	0.26	4.29	65.24	4.46	62.40	7.52	65.45
9	26016	0.59	3.36	54.59	3.60	47.57	5.09	56.35
10	26018	0.48	4.15	77.77	4.70	71.60	9.01	74.17
11	26031	49.37	3.09	27.41	3.41	11.71	4.95	19.39
12	26863	0.17	3.31	85.94	3.36	85.37	9.89	87.02
13	34002	1.95	6.21	65.86	6.47	62.83	10.50	60.89
14	34006	0.71	6.17	60.42	6.40	57.31	10.08	53.08
15	34007	0.81	6.04	66.90	6.20	65.15	10.61	60.86
16	34008	1.91	6.15	54.83	6.32	52.26	9.28	46.98
17	35001	13.52	6.30	81.70	6.39	81.19	15.94	79.78
18	35002	1.3	4.29	74.89	4.87	67.60	9.65	70.24
19	35003	1.27	6.33	83.18	6.32	83.38	16.67	78.47
20	35008	3.68	3.44	79.92	4.19	77.19	7.85	61.77
21	35010	6.35	4.10	79.41	4.21	78.39	8.70	68.09
22	35011	38.36	3.19	46.28	3.88	45.52	4.49	8.1
23	37001	6.76	10.49	59.81	10.58	59.15	17.50	58.68
24	37002	37.23	8.67	62.40	8.67	62.30	13.68	57.12
25	42010	7.97	8.52	75.01	9.01	72.08	19.21	59.76
26	42012	53.42	7.58	73.35	7.72	72.43	15.69	72.12
27	42013	32.33	5.03	83.14	5.09	82.88	15.20	71.56
28	42014	6.6	6.37	70.53	6.72	67.31	12.61	79.79
29	42015	16.19	7.79	72.60	7.87	72.10	15.58	67.57
30	42016	8.42	6.02	74.32	6.19	72.66	12.55	68.4
31	42017	7.53	7.94	76.65	8.03	76.14	17.54	67.1
32	42037	4.57	7.07	75.58	7.01	76.13	14.33	71.66
33	42038	2.27	9.84	69.87	9.87	69.62	20.08	71.24
34	42039	4.01	7.95	68.45	7.94	68.56	15.00	62.77
35	42040	4.57	8.95	63.09	8.90	63.505	15.85	59.58

Table 3 Comparative overall performances of the SCS-CN model, MSCS-CN model and MMSCS-CN model resulted by applications to 35 watersheds

| Sl. No.	Model/procedure	Indices	Mean	Median	Min.	Max.	90 % Confidence interval	
							Lower	Higher
1	SCS-CN	RMSE (mm)	11.39	10.61	2.97	20.08	10.88	12.69
		NSE (%)	61.52	65.45	8.10	87.02	56.89	66.14
2	MSCS-CN	RMSE (mm)	5.96	6.19	2.05	10.58	5.38	6.54
		NSE (%)	65.21	69.62	11.71	85.36	60.56	69.84
3	MMSCS-CN	RMSE (mm)	5.76	6.02	1.97	10.49	5.16	6.36
		NSE (%)	67.94	72.61	27.41	85.94	64.09	71.79

to judge the comparative performance. The expressions for RMSE and NSE are given as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(Q_{obs} - Q_{comp}\right)_i^2} \tag{24a}$$

$$\mathrm{NSE} = \left[1 - \left(\sum_{i=1}^{N} \left(Q_{obs} - Q_{comp}\right)_i^2 \Big/ \sum_{i=1}^{N} \left(Q_{obs} - \overline{Q_{comp}}\right)_i^2\right)\right] \times 100 \tag{24b}$$

where Q_{obs} is observed runoff, Q_{com} and $\overline{Q_{obs}}$ are computed and mean observed runoff, respectively. The efficiency varies on the scale of 0–100. It can also assume a negative value if $\sum\left(Q_{obs} - Q_{com}\right)^2 > \sum\left(Q_{obs} - \overline{Q_{com}}\right)^2$ implying that the variance for observed and computed runoff is greater than the observed data variance. In such a case, the mean of the observed data fits better than does the proposed model.

The efficiency of 100 implies that the computed values are in perfect agreement with observations. EI-Sadek et al. (2001), Fentie et al. (2002) and Michel et al. (2005) used this
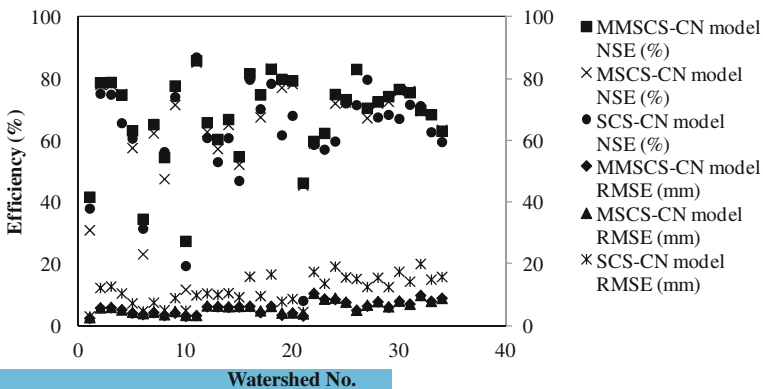

Fig. 1 Efficiency versus RMSE of the MMSCS-CN, MSCS-CN and SCS-CN models
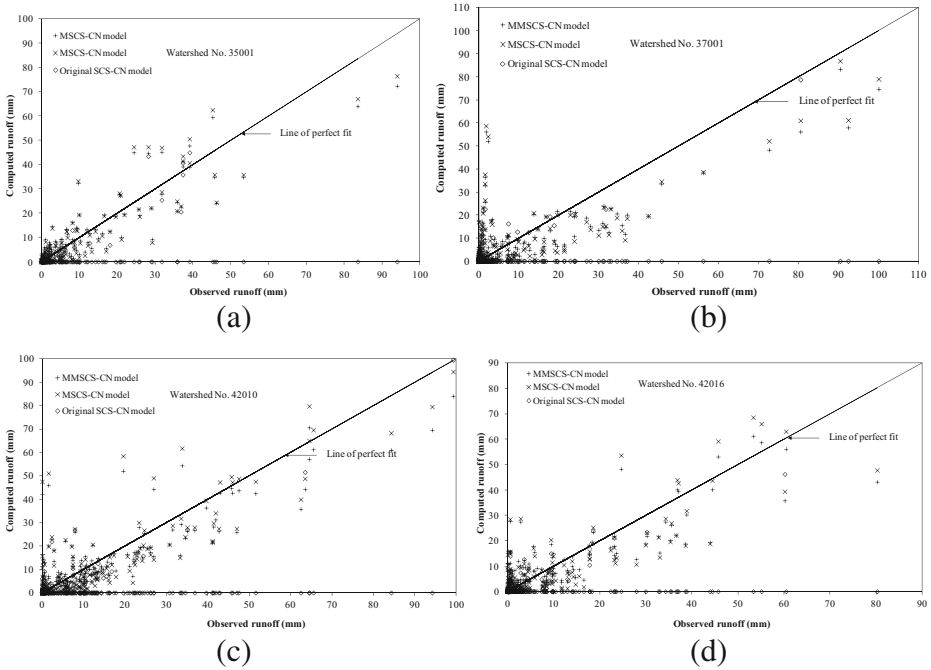
Fig. 2 a–d Comparison between observed and computed runoff by MMSCS-CN, MSCS-CN and SCS-CN models

criterion to compare models performance. McCuen et al. (2006) stated that NSE is a very good criterion for assessing comparative performance of hydrologic models. Here, it is worth emphasizing that higher the NSE, the better is the model performance, and vice versa. Any model having higher NSE and lower RMSE as compared to other models can be rated as improved models and vice versa.

The models performance based on RMSE and NSE resulted by applications to 35 watersheds is given in Table 2. It can be observed from the Table 2 that the MMSCS-CN model always yields higher NSE and lower RMSE as compared to the MSCS-CN and SCS-CN models, and therefore, the previous model can be rated as an improved model as compared to
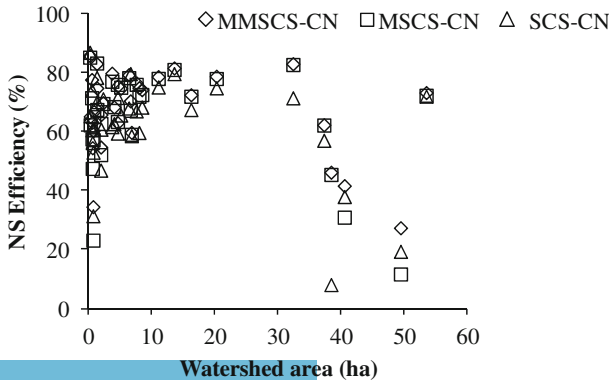
Fig. 3 Comparative performance of MMSCS-CN, MSCS-CN and SCS-CN models based on watershed area

the latter one. Table 3 shows comparative overall performance of the SCS-CN, MSCS-CN and MMSCS-CN models resulted by applications to 35 watersheds. It can be observed from Table 3 that the mean values of RMSE and NSE were found as 11.4, 6.0, 5.8, and 61.5,

Table 4 Goodness-of-fit of models based on NSE and ranking and grading system

Sl. No.	Watershed ID.	Area (ha)	Nash and Sutcliffe (NS) efficiency			Rank and grade based on NSE		
			MMSCS-CN	MSCS-CN	SCS-CN	MMSCS-CN	MSCS-CN	SCS-CN
1	9004	23.96	76.63	74.88	49.09	i(3)	ii(2)	iii(1)
2	16010	40.47	41.71	31.00	38.01	i(3)	iii(1)	ii(2)
3	17001	11.02	78.77	78.15	75.24	i(3)	ii(2)	iii(1)
4	17002	20.21	78.90	78.12	74.95	i(3)	ii(2)	iii(1)
5	17003	5.08	74.83	75.15	65.70	i(3)	ii(2)	iii(1)
6	26010	0.55	63.26	57.63	60.70	i(3)	iii(1)	ii(2)
7	26013	0.68	34.57	23.20	31.49	i(3)	iii(1)	ii(2)
8	26014	0.26	65.24	62.40	65.45	ii(2)	iii(1)	i(3)
9	26016	0.59	54.59	47.57	56.35	ii(2)	iii(1)	i(3)
10	26018	0.48	77.77	71.60	74.17	i(3)	iii(1)	ii(2)
11	26031	49.37	27.41	11.71	19.39	i(3)	iii(1)	ii(2)
12	26863	0.17	85.94	85.37	87.02	ii(2)	iii(1)	i(3)
13	34002	1.95	65.86	62.83	60.89	i(3)	ii(2)	iii(1)
14	34006	0.71	60.42	57.31	53.08	i(3)	ii(2)	iii(1)
15	34007	0.81	66.90	65.15	60.86	i(3)	ii(2)	iii(1)
16	34008	1.91	54.83	52.26	46.98	i(3)	ii(2)	iii(1)
17	35001	13.52	81.70	81.19	79.78	i(3)	ii(2)	iii(1)
18	35002	1.30	74.89	67.60	70.24	i(3)	iii(1)	ii(2)
19	35003	1.27	83.18	83.38	78.47	ii(2)	i(3)	iii(1)
20	35008	3.68	79.92	77.19	61.77	i(3)	ii(2)	iii(1)
21	35010	6.35	79.41	78.39	68.09	i(3)	ii(2)	iii(1)
22	35011	38.36	46.28	45.52	8.10	i(3)	ii(2)	iii(1)
23	37001	6.76	59.81	59.15	58.68	i(3)	ii(2)	iii(1)
24	37002	37.23	62.40	62.30	57.12	i(3)	ii(2)	iii(1)
25	42010	7.97	75.01	72.08	59.76	i(3)	ii(2)	iii(1)
26	42012	53.42	73.35	72.43	72.12	i(3)	ii(2)	iii(1)
27	42013	32.33	83.14	82.88	71.56	i(3)	ii(2)	iii(1)
28	42014	6.60	70.53	67.31	79.79	ii(2)	iii(1)	i(3)
29	42015	16.19	72.60	72.10	67.57	i(3)	ii(2)	iii(1)
30	42016	8.42	74.32	72.66	68.40	i(3)	ii(2)	iii(1)
31	42017	7.53	76.65	76.14	67.10	i(3)	ii(2)	iii(1)
32	42037	4.57	75.58	76.13	71.66	ii(2)	i(3)	iii(1)
33	42038	2.27	69.87	69.62	71.24	ii(2)	iii(1)	i(3)
34	42039	4.01	68.45	68.56	62.77	ii(2)	i(3)	iii(1)
35	42040	4.57	63.09	63.51	59.58	ii(2)	i(3)	iii(1)
Total score						96	63	51
Overall rank						I	II	III

65.2, 67.5, respectively, for the SCS-CN, MSCS-CN, and MMSCS-CN model. The RMSE varied from 3.0–20.8, 2.1–10.6, and 2.0–10.5 mm, respectively, for the SCS-CN, MSCS-CN, and MMSCS-CN model. Similarly, the NSE varied from 8.1–87.0, 11.7–85.4, and 27.4–85.9 %, respectively for the SCS-CN, MSCS-CN, and MMSCS-CN models. It can be observed from Table 3 that the RMSE in case of the SCS-CN model is almost twice as compared to the MMSCS-CN and MSCS-CN models. Further, the MMSCS-CN model always yielded lower RMSE and higher NSE as compared to the MSCS-CN and SCS-CN models, and therefore the proposed MMSCS-CN model appears to be the most suitable alternative as compared to the MSCS-CN and SCS-CN models. Further, for better interpretation of the relative models performance, the NSE and RMSE were plotted as shown in Fig. 1. It can be easily observed from Fig. 1 that the MMSCS-CN model has higher efficiency and lower RMSE as compared to MSCS-CN and SCS-CN models, for most of the study watersheds. Similar inferences can also be drawn from Fig. 2a–d, which show the comparison between observed and computed runoff using all the three models, for the study watersheds. The performance of these models was further assessed on the basis of watershed area. The results are given Fig. 3 and Table 4. It can be observed from Fig. 3 and Table 4 that as the catchment area increases, the MMSCS-CN model performs much better than the rest of the two models, i.e., MSCS-CN and SCS-CN model.

Further, the performance of investigated models was evaluated using the Ranking and Grading System (RGS) Mishra and Singh (1999). The ranks (i) to (iii) were assigned to the models as per their NSE obtained in applications to the data set of each watershed. The rank (i) corresponds to the maximum efficiency and rank (iii) to the minimum. For evaluating the overall performance of these models in all applications, each rank was assigned a grade 3-1 (at an interval of 1, i.e., 3-2-1), respectively, and the assigned grades were added to rank these models in the order of their overall performance. Based on NSE as shown in Table 2, the ranks of models in each application and their overall ranks (I-III) from the overall score obtained by each model are shown in Table 4. It is seen from Table 4 that the MMSCS-CN scores highest mark (95; overall rank I) followed by MSCS-CN with 61 (overall rank II), and SCS-CN model with 51 mark (overall rank III) out of the maximum 105. Based on the overall results obtained, it can be clearly deduced that the MMSCS-CN model is rated as the best model followed by MSCS-CN and SCS-CN model.

9 Conclusions

In this study, an improved SMA based SCS-CN-inspired model (MMSCS-CN) model incorporating a continuous function for antecedent soil moisture was proposed and tested for its suitability over the MSCS-CN and existing SCS-CN models. Specifically, the continuous function for antecedent soil moisture obviates the sudden jumps in runoff estimations and rectifies drawbacks of the SCS-CN and MSCS-CN models. These models were applied to a large data set of rainfall–runoff events derived from 35 US watersheds. Their comparative performance was evaluated using RMSE and NSE values and Ranking and Grading System (RGS). The RMSE varied from 3.0–20.8, 2.1–10.6, and 2.0–10.5 mm, respectively, for the SCS-CN, MSCS-CN, and MMSCS-CN model. Similarly, the NSE varied from 8.1–87.0, 11.7–85.4, and 27.4–85.9 %, respectively for the SCS-CN, MSCS-CN, and MMSCS-CN models. The mean values of RMSE and NSE were found as 11.4, 6.0, 5.8, and 61.5, 65.2, 67.5, respectively, for the SCS-CN, MSCS-CN, and MMSCS-CN model. Finally, using the estimated values of RMSE and NSE, the performance was evaluated using RGS and it was

found that the MMSCS-CN scores highest mark (95; overall rank I) followed by MSCS-CN with 61 (overall rank II), and SCS-CN model with 51 mark (overall rank III) out of the maximum 105. Based on the overall results obtained from this study, it can be concluded that the MMSCS-CN model is the best model followed by MSCS-CN and SCS-CN model.

Appendix A

Equations (14a) & (14b) are re-written here as:

$$V = V_0 + P - \left[\frac{(P + V_0)(P - Ia)}{P + S + V_0} \right] \tag{a1}$$

$$q = p \left[\frac{(P + V_0 + S)(2P + V_0 - Ia) - (P + V_0)(P - Ia)}{(P + S + V_0)^2} \right] \tag{a2}$$

P can be obtained from Eq. (a1) as:

$$P = \frac{[V_0 Ia - (V_0 + S)(V - V_0)]}{(V - V_0) - (S + Ia)} \tag{a3}$$

Using Eq. (a3), the different terms of Eq. (a2) can be written as:

$$(P + V_0 + S) = \frac{S(S + Sa)}{S + Sa - V}, \tag{a4}$$

$$(2P + V_0 - I_a) = \frac{V(S + Sa) - Sa(S + Sa)}{S + Sa - V}, \tag{a5}$$

$$(P + V_0) = \frac{SV}{S + Sa - V}, \tag{a6}$$

$$(P - Ia) = \frac{(S + Sa)(V - Sa)}{S + Sa - V}, \tag{a7}$$

$$\text{and} \qquad (P + S + V_0)^2 = \frac{S^2(S + Sa)^2}{(S + Sa - V)^2} \tag{a8}$$

Using Eqs. (a4)–(a8) into Eq. (a2) and then simplifying results into

$$q = p \left[\frac{VS + (V - Sa)(S_b - V)}{SSb} \right] \tag{a9}$$

This is the desired expression of q and p, i.e., Eq. (15).

Appendix B

Coupling of Eqs. (13) and Eq. (15) results into

$$\frac{dV}{dt} = p\left[1 - \frac{VS + (V-Sa)(S_b-V)}{SSb}\right] \tag{b1}$$

After re-arranging, Eq. (b1) is expressible as:

$$\frac{dV}{dt} = p\left[1 - \frac{V}{S_b} + \left(\frac{V}{S_b} - \frac{S_a}{S_b}\right)\left(1 - \frac{V}{S_b}\right) \Big/ \left(1 - \frac{S_a}{S_b}\right)\right] \tag{b2}$$

or

$$\frac{dV}{dt} = p\left[\frac{(S_b-V)^2}{SSb}\right] \tag{b3}$$

Again, re-arranging Eq. (b3) and applying appropriate lower and upper limits of integration results into

$$\int_{V=V0}^{V} \frac{dV}{(S_b-V)^2} = \int_{t=0}^{t} \frac{pdt}{SSb} \tag{b4}$$

On integrating Eq. (b4), we get

$$\frac{1}{(S_b-V)} - \frac{1}{(S_b-V_0)} = \frac{P}{SSb} \tag{b5}$$

Now, substituting V from Eq. (12) into Eq. (b5) and rearranging leads to

$$\frac{(P-Q)}{[(S_b-V_0)-(P-Q)](S_b-V_0)} = \frac{P}{SSb} \tag{b6}$$

Equation (b6) can be further simplified as:

$$Q = P\left(1 - \frac{(S_b-V_0)^2}{SSb + P(S_b-V_0)}\right) \tag{b7}$$

.This is the desired expression of Q and P, i.e., Eq. (19).

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